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SYNTHESIS OF THE AUTOMATIC REGULATION SYSTEM FOR THE RELATED OBJECTS

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Abstract. In the paper the possibility of synthesis of the autonomous control system is investigated with the known transmission function of the main channels. Based on these transmission functions and invariance conditions, the transmission functions for the compensators are obtained. A functional diagram of autonomous control systems of the second order is constructed. A static autonomous control system for the third-order systems is developed. The solution of the specific example for systems of the third order is considered. Simulation systems were built on MATLAB/Simulink.

Keywords: Transmission function, related systems, autonomy, invariance, separate channel, compensator, Simulink.

AMS Subject Classification: C13, C22, F31.

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1 Introduction

In this paper, the concept of separate is used in the sense of "independent". When synthesizing multidimensional control systems, the presence of static and dynamic connections between the inputs and outputs of the object significantly complicates the solution of the problem (Besekerskiy, 2003; Vostrikov & Fransuzova, 2023). As is known from the literature such kind of problems have a broad range of applications and are investigated from different of aspects (Shen et al., 2021; Tadewos et al., 2022; Zardini et al., 2021; Fraser, 2020).

To simplify the synthesis problem, we accept the following assumptions (Aliyev, 1993; Lazareva et al., 2004):

1. The number (dimension) of control inputs ui is equal to the number of outputs dim u=dim y this means that each output can be controlled by the corresponding input;
2. Cross links are directed in one direction from input to output;
3. The number of inputs and inputs is 2, i=1.2. This means that the input and output of the object is two-dimensional.

Figure 1 shows the information diagram of a connected object.

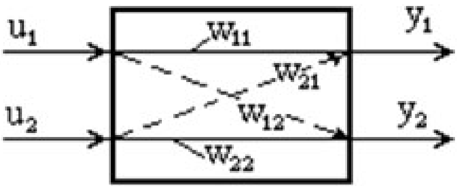


Figure 1: Scheme of the connected object. Here W_{11},W_{22} and W_{12},W_{21} are the transmission functions on the main and cross channels correspondingly.

The quality of coupled systems can be improved by applying various methods of compensating for cross-links. The most effective of the compensation methods is based on the principle of autonomy. Autonomy consists in independence in dynamics and statics from each other of two output variables $y1$ and $y2$ of the system.

In fact the separate consists of two conditions of invariance: the first is independence (invariance) of $y1$ from setting $g2$ of the second control system; the second is the independence of $y2$ from the setting $g1$ of the first control system.

In this case, the signal $g1$ for $y2$, and $g2$ for $y1$ is considered as a perturbing action. This means that the cross channels play the role of perturbations. To compensate for the action of these disturbing forces, dynamic compensators with transfer functions and are introduced into the system (see Fig. 1). Depending on the application point of the compensator output, there can be two configurations:

1. Can be transferred to other control channels.

2. Can be fed to the inputs of regulators.

Figure 2 shows a diagram of an autonomous system corresponding to the first case.

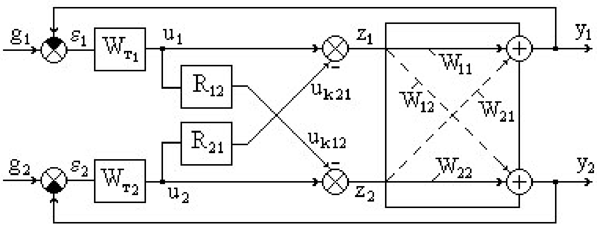


Figure 2: Transmission of output signals of compensators to other control channels.

In the Figures, the transmission functions W_{B_1},W_{B_2} can be selected as standard P, PI and PID controllers. The tuning parameters of these controllers after fulfillment of the conditions of autonomy can be calculated by one of the methods of parametric synthesis of single-loop automatic control systems MCS.

Consider the definitions of transmission functions $R_{12}(s)$ and $R_{21}(s)$ and compensators from the separate condition.

Based on the scheme shown in Fig. 2, we determine the images of the output values y_1:

$$Y_1 = W_{11}z_1 + W_{21}z_2. \tag{1}$$

Based on schema intermediate variables z_1 are z_2

$$z_1 = (G_1 - Y_1)W_{B1} - (G_2 - Y_2)W_{B2}R_{21} \tag{2}$$

$$z_2 = (G_2 - Y_2)W_{B2} - (G_1 - Y_1)W_{B1}R_{12} \tag{3}$$

Substituting expressions (1) and (2) into (3) and grouping, we can write

$$Y_1 = (W_{B1}W_{11} - W_{B1}R_{12}W_{21})(G_1 - Y_1) + \\ +(W_{B2}W_{21} - W_{B2}R_{21}W_{11})(G_2 - Y_2) \ .$$

From the obtained expression as an equality, in order that the existence of g_2 did not affect the output y_1, the invariance (independence) condition below should be met

$$W_{B2}W_{21} - W_{B2}R_{21}W_{11} = 0.$$

From here we determine the transfer function of the compensator

$$R_{21}(s) = \frac{W_{21}(s)}{W_{11}(s)}. \tag{4}$$

As one can see, the transmission function of the compensator depends on the ratio of the transfer functions of the cross and direct channels of the object.

Similarly, for the compensator R_{12} we have

$$R_{12}(s) = \frac{W_{12}(s)}{W_{22}(s)}. \tag{5}$$

As can be seen from the system, the transfer functions of the compensators built in Fig. 2, are determined based on the known transmission functions of the object. This feature does not allow one to interfere with the choice of transmission functions of compensators. This is the negative side of the technique.

Figure 3 shows the Automatic Control System (ACS), when the output signals of the compensators are given to the inputs of the regulators. In this case we have

$$R_{21}(s) = \frac{W_{21}(s)}{W_{11}(s)} \cdot \frac{1}{W_{B1}}, \ \ R_{12}(s) = \frac{W_{12}(s)}{W_{22}(s)} \cdot \frac{1}{W_{B2}}.$$

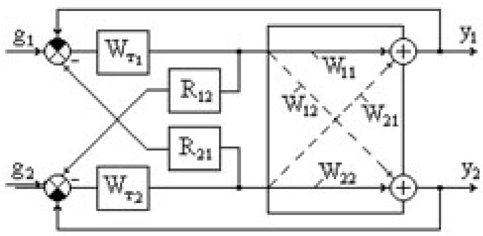


Figure 3: Supply of output signals of compensators at the input of regulators

As can be seen from Figure 3 the transfer functions of the compensators also include the transfer functions of the controllers. This increases their independence. Nevertheless, the regulators should be chosen in such a way that, simultaneously with the improvement of the quality, the condition of the physical feasibility of the compensators is also fulfilled. So that the regulators can be chosen in such a way, simultaneously with the improvement of the quality indicators of autonomous channels, the condition of the physical feasibility of the compensators was fulfilled.

Like combined systems and in autonomous control systems, in technical implementation, the physical feasibility of the transfer functions of compensators plays an important role. If there is a dissatisfaction with the conditions of autonomy, a decrease in stability margins, an increase in oscillation, and similar undesirable processes, then known static or asymptotic methods can be used to compensate these issues. In this case, during transitions (dynamic mode), the influence of cross-links remains. Only when the system approaches the steady state, this influence is eliminated and only one output receives the value of the corresponding output. Such systems are called static autonomous systems.

If the astatic PI- and PID-regulators are used as the main regulators, then there is no need to use a static compensator. Since in steady state these regulators compensate for constant references.

2 Static autonomous systems

In the case when the main regulators are proportional P-regulators $W_{B1} = k_1$,$W_{B2} = k_2$ let's consider the solution of the problem of synthesis of a static compensator. Due to the simplicity of this task, we will use the generalized scheme given in Fig.4.

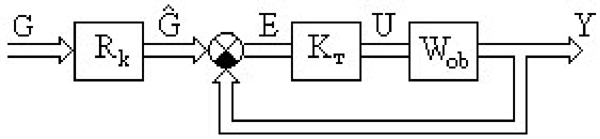


Figure 4: Generalized static autonomous ACS

Here $G = (G_1, \ldots, G_n)^B$ is the vector real tasks; $Y = (Y_1, \ldots, Y_n)^B$ is the vector of controlled values; $K_B = \begin{pmatrix} k_1 & 0 \\ 0 & k_n \end{pmatrix}$ is the matrix of diagonal elements of the P-controller; $W_{ob} = \begin{pmatrix} w_{11} & \cdots & w_{1n} \\ & \ldots\ldots\ldots\ldots & \\ w_{n1} & \cdots & w_{nn} \end{pmatrix}$ is the object transmission matrix; $R_k = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ & \ldots\ldots\ldots & \\ r_{n1} & \cdots & r_{nn} \end{pmatrix}$ is the matrix of seeking compensator elements.

Based on Figure 4 the open-loop equation is obtained as

$$Y(s) = W_{ob}(s)K_B E(s).$$

If this expression to consider the error as

$$E(s) = \hat{G}(s) - Y(s),$$

then we get the following equation of the closed-loop system

$$Y(s) = [I + W_{ob}(s)K_B]^{-1} W_{ob}(s)K_B\hat{G}(s). \tag{6}$$

Here I is uniqiue matrix of dimesion $n \times n$.

To take into account the actual error in (6), it is necessary to write the expression $\hat{G} = R_k G$. Then

$$Y(s) = [I + W_{ob}(s)K_B]^{-1} W_{ob}(s)K_B R_k G(s). \tag{7}$$

In statics, to fulfill the condition of autonomy, i.e. the impact of each g_i on the corresponding y_i, $i = 1, n$ at $s \to 0$ ($t \to \infty$) in (7) the coefficient G must be diagonal, or in a special case, the identity matrix. In this case, the following condition must be satisfied

$$\lim_{s\to 0} [I + W_{ob}(s)K_B]^{-1} W_{ob}(s)K_B R_k = I = \begin{pmatrix} 1 & \ldots & 0 \\ & \ldots\ldots & \\ 0 & \ldots & 1 \end{pmatrix}. \tag{8}$$

From relation (8) we determine the gain matrix R_k

$$R_k = \lim_{s\to 0} \left[(I + W_{ob}K_B)^{-1} W_{ob}K_B\right]^{-1}. \tag{9}$$

From the last we obtain

$$R_k = [W_{ob}(0)K_B]^{-1}\,[I + W_{ob}(0)K_B] = [W_{ob}(0)K_B]^{-1} + I, \tag{10}$$

where $[W_{ob}(0)K_B]^{-1} = K_B^{-1}W_{ob}^{-1}(0)$.

Example. Let the transmission matrix of the three-dimensional object in terms of input and output be given in the form

$$W_{ob}(s) = \begin{bmatrix} \frac{0.7}{9s+1} & 0 & 0 \\ \frac{2.0}{8s+1} & \frac{0.4}{6s+1} & 0 \\ \frac{2.3}{1+10s} & \frac{2.3}{8s+1} & \frac{2.1}{7s+1} \end{bmatrix}$$

and the transmission matrix of the P-controller is given as

$$K_B = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}.$$

The gain matrix of the static compensator is determined by expression (10). Since the inverse matrices of the controller and plant are as follows

$$K_B^{-1} = \begin{bmatrix} 1/k_1 & 0 & 0 \\ 0 & 1/k_2 & 0 \\ 0 & 0 & 1/k_3 \end{bmatrix},$$

$$W_{ob}^{-1}(0) = \begin{bmatrix} 1.43 & 0 & 0 \\ -7.14 & 2.5 & 0 \\ 6.26 & -2.74 & 0.48 \end{bmatrix}$$

then, based on expression (10), we determine

$$R_k = \begin{bmatrix} \frac{1.43}{k_1} + 1 & 0 & 0 \\ \frac{-7.14}{k_2} & \frac{2.5}{k_2} + 1 & 0 \\ \frac{6.26}{k_3} & \frac{-2.74}{k_1} & \frac{0.48}{k_1} + 1 \end{bmatrix}. \tag{11}$$

As can be seen from expression (11), as the gains of the controllers $k_i \to \infty$, $i = 1,\ 2,\ 3$ increase, the matrix $R_k \to I$, in other words, the gains of the compensator, approach to 1. If it is possible to use large gains, a non-astatic P-controller can also be used.

Figure 5 shows a diagram of a static autonomous control system.

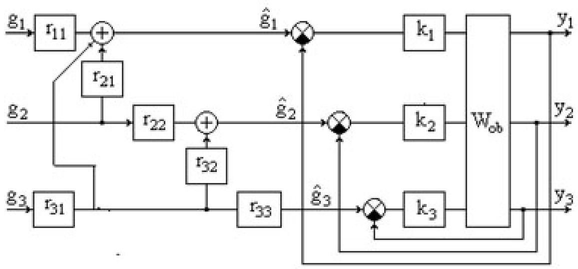
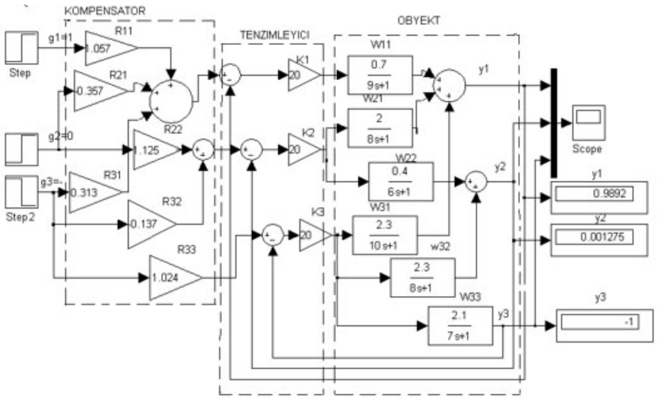
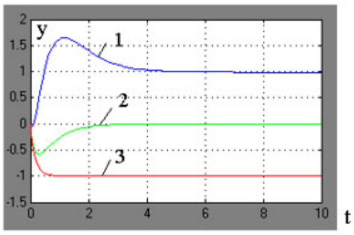


Figure 5: Static autonomous system

In Figure 6: a) and b) when setting $g1 = 1, g2 = 0, g3 = -1$ and controller gains $ki = 20, i = 1 - 3$ shows the scheme on Simulink (a) and transision responses $y1, y2, y3$ (b).



a)



b)

Figure 6: Simulink diagram of autonomous ACS (a) and transision (b)

It can be seen from the figure that, despite the existence of cross-links on the object, after the completion of transient processes, the controlled values $y1, y2$ and $y3$ are equal to their assignments.

If to use the astatic PI controllers in static mode, the setpoint could be reached without error. In practice, autonomous systems do not have high efficiency. If there are delays in the cross channels, then the synthesis must be performed taking these delays into account. For this, you can use the Smith prognosticator.

One of the important features is that if the object model (transfer function) differs significantly from the real model, then the quality of regulation can deteriorate significantly, even a closed system can become unstable.

3 Conclusion

The paper analyzes the patterns of fulfillment of autonomy conditions in one-dimensional and three-dimensional coupled control systems. Transition equations and limit processes in coupled systems are obtained in vector-matrix form. The problem of applied control for 3rd order systems is solved. Corresponding functional schemes have been developed on Matlab/Simulink. As a result, the transition characteristics of the coupled system are constructed, which confirm the reliability of the theoretical results.

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